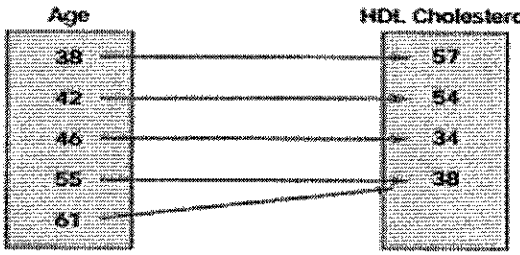
4.2 – One to One Functions and Inverse Functions

- A function is <u>one-to-one</u> if any two different inputs in the domain correspond to two different outputs in the range.

- A function is NOT <u>one-to-one</u> if two different inputs correspond to the same output.

Determine whether the following functions are one-to-one.

(a) For the following function, the domain represents the age of five males and the range represents their HDL (good) cholesterol (mg/dL).



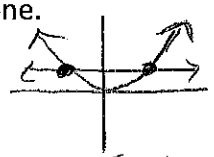
NOT one-to-one b/c 2 different inputs go to the same output.

(b) $\{(-2, 6), (-1, 3), (0, 2), (1, 5), (2, 8)\}$

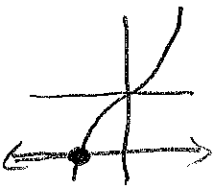
Yes, it is one-to-one, b/c every input (x-coordinates) has only one output (y-coordinates)

<u>Horizontal Line Test</u> – If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

(crosses at 2 places)



NOT one-to-one

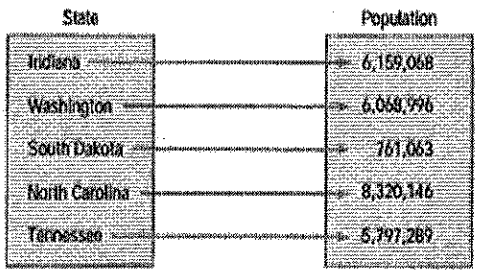


(crosses at 1 spot)

one-to-one

<u>Finding the Inverse of a function defined by a map</u>

Find the inverse of the following function. Let the domain of the function represent certain states, and let the range represent the state's population. State the domain and the range of the inverse function.



(Domain) Population

(Range) STATE

6,159,068 → Indiana
6,068,996 → Washington
761,063 → South Dakota
8,320,146 → North Carolina
5,797,289 → Tennessee

<u>Finding the inverse of a function defined by a set of ordered pairs</u>

Find the inverse of the following one-to-one function:

$\{(-3, -27), (-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8), (3, 27)\}$

* switch the x & y coordinates

$\{(-27, -3), (-8, -2), (-1, -1), (0, 0), (1, 1), (8, 2), (27, 3)\}$

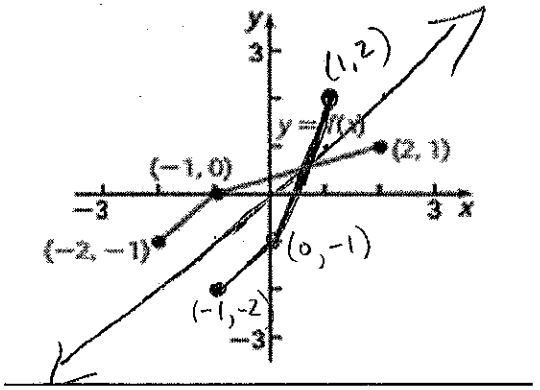
** Domain of f = Range of f^{-1} Range of f = Domain of f^{-1}

Verifying Inverse Functions

Verify that the inverse of $f(x) = \frac{1}{x-1}$ is $f^{-1}(x) = \frac{1}{x} + 1$. For what values of x is $f^{-1}(f(x)) = x$? For what values of x is $f(f^{-1}(x)) = x$?

- $f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x-1}\right) = \frac{1}{\left(\frac{1}{x-1}\right)} + 1 = x - 1 + 1 = (x)$, $x \neq 1$
- $f(f^{-1}(x)) = f\left(\frac{1}{x} + 1\right) = \frac{1}{\left(\frac{1}{x}+1\right)-1} = \frac{1}{\frac{1}{x}} = (x)$, $x \neq 0$

The following graph is one-to-one. Draw the graph of its inverse



x	y
−2	−1
−1	0
2	1
−1	−2
0	−1
1	2

→ Plot these values, they are the inverses of the given points

* If a graph is one-to-one, then its inverse should be a reflection across the line $y = x$.

Finding the Inverse Function

Find the inverse of $f(x) = 2x + 3$. Also find the domain and range of f and f^{-1}. Graph f and f^{-1} on the same coordinate axes.

Steps for finding the Inverse function

(1) rewrite the function, flip flopping the x's & y's

$x = 2y + 3$

(2) Solve for y ⟹ $\frac{x-3}{2} = \frac{2y}{2}$ → $f^{-1}(x) = \frac{x-3}{2}$

Domain & Range of both f & f^{-1} are ℝ b/c both functions are linear

The function

$$f(x) = \frac{2x+1}{x-1}, \quad x \neq 1$$

is one-to-one. Find its inverse and check the result.

$y = \frac{2x+1}{x-1}$

(1) switch x & y, then multiply each side by $(y-1)$

$(y-1)x = \frac{2y+1}{y-1}(y-1)$

(2) distribute, combine like terms, move any term w/a y in it to the Left & everything else to the right.

(3) Factor out a y, & solve for y

$xy - x = 2y + 1$

$xy - 2y = x + 1$

$\frac{y(x-2)}{x-2} = \frac{x+1}{x-2}$

$f^{-1} = \frac{x+1}{x-2}$